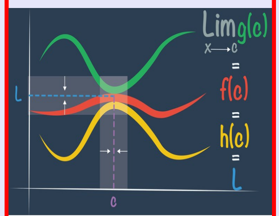
Calculus I

Lecture 17



Feb 19-8:47 AM

Evaluate $\lim\limits_{x\to 0} \dfrac{\sin 5x}{4x}$ $\quad \dfrac{\sin 0}{0} = \dfrac{0}{0}$ I.F.

$$= \frac{1}{4} \lim_{x\to 0} \frac{5\sin 5x}{5x}$$

$$\lim_{h\to 0} \frac{\sin h}{h} = 1$$

$$= \frac{1}{4} \cdot 5 \boxed{\lim_{x\to 0} \frac{\sin 5x}{5x}} = \frac{1}{4} \cdot 5 \cdot 1 = \boxed{\frac{5}{4}}$$

Evaluate $\lim\limits_{x\to 0} \dfrac{\sin 3x}{5x^3 - 4x}$ $\quad \ldots \dfrac{0}{0}$ I.F.

$$\lim_{x\to 0} \frac{\sin 3x}{5x^3 - 4x} = \lim_{x\to 0} \frac{\sin 3x}{x(5x^2-4)} = \lim_{x\to 0} \left[\frac{3\sin 3x}{3x} \cdot \frac{1}{5x^2 - 4}\right]$$

$$= 3 \boxed{\lim_{x\to 0} \frac{\sin 3x}{3x}} \cdot \lim_{x\to 0} \frac{1}{5x^2-4} = 3 \cdot 1 \cdot \frac{1}{-4} = \boxed{\frac{-3}{4}}$$

Sep 25-7:28 AM

Evaluate $\lim_{x \to 0} \frac{\sin 3x \sin 5x}{x^2}$ $\frac{0}{0}$ I.F.

$$= \lim_{x \to 0} \left[\frac{3 \sin 3x}{3x} \cdot \frac{5 \sin 5x}{5x} \right]$$

$$= 3 \cdot \cancel{\lim_{x \to 0} \frac{\sin 3x}{3x}}^{\to 1} \cdot 5 \cancel{\lim_{x \to 0} \frac{\sin 5x}{5x}}^{\to 1}$$

$$= \boxed{15}$$

Sep 25-7:36 AM

Evaluate $\lim_{x \to 0} \frac{\sin x}{x + \tan x}$ ... $\frac{0}{0}$ I.F.

$$= \lim_{x \to 0} \frac{\sin x}{x + \sin x \cdot \frac{1}{\cos x}} = \lim_{x \to 0} \frac{\frac{\sin x}{x}}{\frac{x}{x} + \frac{\sin x}{x} \cdot \frac{1}{\cos x}}$$

$$= \frac{\lim_{x \to 0} \frac{\sin x}{x}}{\lim_{x \to 0} \frac{x}{x} + \lim_{x \to 0} \frac{\sin x}{x} \cdot \lim_{x \to 0} \frac{1}{\cos x}} = \frac{1}{1 + 1 \cdot \frac{1}{1}}$$

$$= \boxed{\frac{1}{2}}$$

Sep 25-7:42 AM

Evaluate $\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{\sin x - \cos x} = \frac{1 - \tan\frac{\pi}{4}}{\sin\frac{\pi}{4} - \cos\frac{\pi}{4}} = \frac{1-1}{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}} = \frac{0}{0}$

$\tan\frac{\pi}{4} \to 1$, $\sin\frac{\pi}{4} \to \frac{\sqrt{2}}{2}$, $\cos\frac{\pi}{4} \to \frac{\sqrt{2}}{2}$

I.F.

$$\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{\sin x - \cos x} = \lim_{x \to \frac{\pi}{4}} \frac{1 - \frac{\sin x}{\cos x}}{\sin x - \cos x}$$

$\frac{a-b}{b-a} = -1$

multiply by $\cos x$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cancel{\cos x - \sin x}}{\cos x(\cancel{\sin x - \cos x})}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{-1}{\cos x} = \frac{-1}{\cos\frac{\pi}{4}} = \frac{-1}{\frac{\sqrt{2}}{2}}$$

$$= -\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{-\sqrt{2}}$$

Sep 25-7:48 AM

Use ε and δ definition to prove

$$\lim_{x \to -2} (-5x + 4) = 14$$

$f(x) = -5x + 4$

$a = -2$

$L = 14$ ✓

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|-5x + 4 - 14| < \varepsilon$ $\quad$ $|x - (-2)| < \delta$

$|-5x - 10| < \varepsilon$

$|-5(x+2)| < \varepsilon$

$|-5|\,|x+2| < \varepsilon$

$\to 5|x+2| < \varepsilon$

$|x+2| < \frac{\varepsilon}{5}$

$\boxed{\text{Pick } \delta = \frac{\varepsilon}{5}}$

Sep 25-7:55 AM

Use ε and δ def. to prove $\lim_{x\to 1} x^4 = 1$.

$f(x) = x^4$, $a = 1$, $L = 1$ ✓

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|x^4 - 1| < \varepsilon$ " $|x - 1| < \delta$

$|(x^2+1)(x^2-1)| < \varepsilon$ " $|x - 1| < \delta$

$|x^2+1|\,|x+1|\,|x-1| < \varepsilon$

Bound Keep

If $\delta \le 1$, then

$|x-1| < 1$

$-1 < x - 1 < 1$

$0 < x < 2$

If $x = 0^+$ $|x^2+1|\,|x+1| > 1$

If $x = 2^-$ $|x^2+1|\,|x+1| < 15$

$\Rightarrow 1 < |x^2+1|\,|x+1| < 15$

$|x^2+1|\,|x+1|\,|x-1| < 15\,|x-1| < \varepsilon$

$|x-1| < \frac{\varepsilon}{15}$

Pick $\delta = \min\left\{1, \frac{\varepsilon}{15}\right\}$

Sep 25-8:06 AM

Class Quiz 7

Use $\boxed{m_{\substack{\text{tan. line}\\ \text{at } x=a}} = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}}$ to find

slope of the tan. line to the graph of

$\boxed{f(x) = \frac{1}{x}}$ at $\boxed{x = a}$, $a \ne 0$.

$$m = \lim_{x\to a} \frac{f(x) - f(a)}{x - a} = \lim_{x\to a} \frac{\frac{1}{x} - \frac{1}{a}}{x - a} = \lim_{x\to a} \frac{xa\cdot\frac{1}{x} - xa\cdot\frac{1}{a}}{xa(x-a)}$$

LCD $= xa$

$$= \lim_{x\to a} \frac{a - x}{xa(x-a)} = \lim_{x\to a} \frac{-1}{xa} = \boxed{\frac{-1}{a^2}}$$

Sep 25-8:16 AM